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Common myths on yield to maturity in bonds or IRR in corporate finance

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SUMARIO

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Resumen

El rendimiento al vencimiento (YTM) o la tasa interna de rendimiento (TIR) es una métrica utilizada en el análisis financiero para estimar la rentabilidad de las inversiones potenciales. Casi todos los libros de texto de finanzas establecen los siguientes supuestos condicionantes: (i) que los pagos del cupón se pueden reinvertir a una tasa igual al rendimiento hasta el vencimiento, (ii) que el bono se mantiene hasta el vencimiento. Mostramos que hay dos falacias comunes acerca de estos supuestos, y ninguno de ellos es necesario para interpretar esta medida de retorno, y probablemente hayan surgido como consecuencia de un malentendido semántico. El cálculo del YTM / IRR es el resultado de una operación matemática ex ante que se centra en los flujos de caja actuales y futuros, independientemente de la tasa de reinversión, que es diferente de la acumulación de riqueza. Al final del artículo proporcionamos algunos ejemplos numéricos.

Abstract

The yield to maturity (YTM) or internal rate of return (IRR) is a metric used in financial analysis to estimate the profitability of potential investments. Almost all finance textbooks state the following conditioning assumptions: (i) that the coupon payments can be reinvested at a rate equal to the yield to maturity, (ii) that the bond is held to maturity. We show that there are two common fallacies about it these assumptions, and none of them are necessary to interpret this return measure, and they may have probably arisen as a consequence of a semantic misunderstanding. The calculation of the YTM/IRR is the result of an ex ante mathematical operation focusing on current and future cash flows, regardless of the reinvestment rate, which is different from wealth accumulation. At the end of the paper we provide some numerical examples.

1. Introduction

The Yield to Maturity (YTM) or Internal Rate of Return (IRR) is an ex-ante metric used in financial analysis to estimate the profitability of potential investments. This metric represents a relationship between cash flows at different times, expressed as the average geometric periodic rate of return that satisfies the constraint that the Net Present Value of all cashflows should be zero, i.e. the first term of the left hand side of the following equation (showing the current investment cost or purchase price of the investment) should be mathematically equal to the second term (showing future cash flows from the investment), by changing and obtaining a value for rate IRR (standing for YTM or IRR). [1]

[1] The formula =IRR() or in spanish =TIR()

$$0 = -I_0 + \sum_{t=1}^{t=n} \frac{FF_t}{(1 + IRR)^t} \qquad \textit{Eq 1}$$

Because of the nature of the formula, the value IRR cannot be easily calculated analytically and must therefore be calculated either through trial-and-error which can be easily done in Excel.

A higher rate IRR usually stands for a more attractive investment under the same risk conditions and the same maturity and can be used to rank multiple alternative investments or projects on a relatively even basis. A higher IRR means the investor can achieve higher expected cashflows in the future.

Mathematically, one can think of rate IRR as the annually compound average rate of growth of an investment (CAGR), hence it is an average of returns considering all periods and cashflows, or, in other words, it is a standardisation of returns to facilitate comparison of investments with multiple irregular cashflow payoffs.

There are three possible interpretations of rate IRR (YTM), depending on the certainty of the cash flows involved:

a) riskless rate of return, if the cash flows are certain
b) promised rate of return, if the cash flows are contractual promises but not expected
b) expected rate of return, if the cash flows are projections

In the case of certainty, as with riskless bonds, the investment is the purchase or market price and the returns are the payments of promised coupons and principal. The calculation is carried out by iteration seeking the rate that equates cash outflows to cash inflows. No assumptions are needed for this.

We differentiate between the cash flows from a bond (or project) from the cash flows arising from the owners' strategy with those receipts. The following cases apply both for a bond, or a corporate finance investment project, since their analysis in both cases resume to analysing a vector of future cashflows.

2. Certainty: Risk free bonds

Assume a 10 year bullet bond promising a 9 yearly coupon priced at par (100). The YTM or IRR of the cash flows from the bond is 9%. Its duration is 7 years. Therefore any investor buying the bond at time 0 and collecting all the coupons and principal will have placed the 100 at a 9% yield to maturity.

An investor carrying out a strategy to reinvest all the coupons in the same or a twin bond until the final maturity date who is able to do so at the same rate as the original 9% will receive an amount equal to 100 x 1.09^{10} = 236.74 cash flow at the final maturity time 10.

Table 1: Cash flows: reinvesting all coupons

t	bond	reinvestment	investor
0	(100,00)		-100,00
1	9,0	(9,0)	0,0
2	9,0	(9,0)	0,0
3	9,0	(9,0)	0,0
4	9,0	(9,0)	0,0
5	9,0	(9,0)	0,0
6	9,0	(9,0)	0,0
7	9,0	(9,0)	0,0
8	9,0	(9,0)	0,0
9	9,0	(9,0)	0,0
10	109,0	127,7	236,7
IRR	9,0%	9,0%	9,0%

The cash at time 10 is the value of all the bonds purchased with the coupons plus principal. Both cash flows under column 1, the bond's, and column 3, the investor's have same the same IRR, regardless of the strategy of the bond's holder.

Alternatively, assume an investor who invests half of the coupons. At time 10 the investor's fund will be less than that of the full coupon reinvestment strategy, but would have received net cash flows of 4.5 during the life of the bond. The rate of return, IRR, of the strategy is 9%. The investor's strategy, independent of the bond's cash flows is:

Table 2: Partially reinvesting coupons

t	bond	reinvestment	investor
0	(100.00)		(100.00)
1	9.0	(4.5)	4.5
2	9.0	(4.5)	4.5
3	9.0	(4.5)	4.5
4	9.0	(4.5)	4.5
5	9.0	(4.5)	4.5
6	9.0	(4.5)	4.5
7	9.0	(4.5)	4.5
8	9.0	(4.5)	4.5
9	9.0	(4.5)	4.5
10	109.0	63.9	172.9
IRR	9.0%	9.0%	9.0%

The IRR of the bond is independent of the countless strategies an investor may execute. The investor's strategy rate of return depends on the rate of return of the bond.

An investor deciding to accumulate wealth will reinvest more by moving cash flows received from coupons to later dates. *This changes the duration of the investment*. If full reinvestment until final maturity is made the duration on the strategy will be higher than the duration of the bond, 10 years in our example, or 8.5 years if half of all the coupons are re invested until final maturity.

An alternative strategy is reinvesting fully up to a certain date, or horizon H, and selling the fund. With unchanged rates, if the fund is sold at any time before maturity, it will return the IRR. This breaks assumption (ii) “it is assumed that the bond is held to maturity”.

Table 3: Fund value at different times with unchanged rate

t	investor	reinvest	value reinv.	bond value	fund value
0	(100.00)				
1	0.0	(9.00)	9.00	100.00	109.00
2	0.0	(9.00)	18.81	100.00	118.81
3	0.0	(9.00)	29.50	100.00	129.50
4	0.0	(9.00)	41.16	100.00	141.16
5	0.0	(9.00)	53.86	100.00	153.86
6	0.0	(9.00)	67.71	100.00	167.71
7	0.0	(9.00)	82.80	100.00	182.80
8	0.0	(9.00)	99.26	100.00	199.26
9	0.0	(9.00)	117.19	100.00	217.19
10	236.7	127.74	127.74	109.00	236.74

However, rates change. Assume an investor decides to fully re invest coupons until a horizon H, equal to the date of the duration D of the bond, so that H = D. If rates do not change, at time D the strategy will have accrued a fund value of 182.80, composed by the bond and the reinvested coupons. If rates change instantaneously after the bond’s purchase, dropping from 9% to 8%, two things happen: a) the bond's price rises, and b) the re investments earn less:

Table 4: The value of fund at different rates

rates drop to 8%				*rates rise to 10%*			
t	value of reinvestments	value bond	fund value	t	value of reinvestments	value bond	fund value
1	9.00	106.25	115.25	1	9.00	94.24	103.24
2	18.72	105.75	124.47	2	18.90	94.67	113.57
3	29.22	105.21	134.42	3	29.79	95.13	124.92
4	40.56	104.62	145.18	4	41.77	95.64	137.41
5	52.80	103.99	156.79	5	54.95	96.21	151.16
6	66.02	103.31	169.34	6	69.44	96.83	166.27
7	80.31	102.58	182.88	7	85.38	97.51	182.90
8	95.73	101.78	197.51	8	102.92	98.26	201.19
9	112.39	100.93	213.31	9	122.22	99.09	221.31
10	121.38	109.00	230.38	10	134.44	109.00	243.44

At time t=7 the value of the fund will be 182.88 or 182.90 in either case, which, if sold, will yield slightly more than 9%, the IRR of the bond at purchase. Whatever the reinvestment rate, the fund at duration D will yield the at least the IRR. This breaks assumption (i) “it is assumed that the coupon payments can be reinvested at a rate equal to the yield to maturity”.

This analysis can be extended to multiple changes in interest rates through interest rate immunisation strategies, consisting in maintaining the duration of the bond portfolio equal to the horizon target. This implies rebalancing as interest rates change.

The cause of the confusion arisen from the “assumptions” of YTM or IRR appears to be semantic. The “maturity” of a set of multi periodic cash flows is ambiguous: there are multiple maturities starting with the date of the payment of the first coupon through to the payment of the last cash flow. Maturity as a convention is generally interpreted as the date of the last payment, but cash flows occur before that date. Duration is a better measure of maturity because it is a weighted average of all dates when cash flows occur, not just the final date. In our example of a 10 year bond, this can be shown as how much of the investment is made at different maturities.

Table 5: PV of bond cash flows as % of investment

t	cash flow	PV[CF]	investment up to different dates
1	9	8.26	8.3%
2	9	7.58	7.6%
3	9	6.95	6.9%
4	9	6.38	6.4%
5	9	5.85	5.8%
6	9	5.37	5.4%
7	9	4.92	4.9%
8	9	4.52	4.5%
9	9	4.14	4.1%
10	109	46.04	46.0%

The amount of the bond price invested in the last payment - principal and coupon 10, is 46%, with the remaining 54% invested at sooner dates. Maturity is all of those dates.

3. Uncertainty

a) Promised cash flows: Credit risky bonds

The yield to maturity of risky bonds is the rate of return that compares price to promised payments. It is the maximum rate of return to be earned if default does not materialise. Lower, even negative, realised returns may happen.

Therefore scenarios where cash flows will be lower and or later than promised are possible and, as a consequence, a strategy aimed to accumulate wealth may not be successful. Only if default begins at a date later than duration, and if the bond plus re investments are sold at duration, a strategy aimed at immunising IRR may prove successful.

Promised yield to maturities of risky bonds are not comparable to riskless yields. Their spreads on riskless assets reflect the market discount due to risk perceptions, not higher expected returns

b) Expected cash flows: projects

An YTM/IRR computed with uncertain projected cash flows is an expected YTM/IRR. The uncertainty on the timing and size of the cash flows accumulate on a planned reinvestment strategy. Expected YTM/IRRs are comparable to risk free rates of return provided they are adjusted for risk.

c) The difference between MIRR and IRR

YTM and IRR are popular metrics among investors, but it is said that YTM/IRR tends to overstate the profitability of an investment opportunity since it requires that the cashflows received shall be reinvested at the same rate, hence if that is not possible, it may lead to overstate the true yield of the investment opportunity.

It is also said that the modified internal rate of return (MIRR) compensates for this inconvenience by incorporating into the analysis the chance of evaluating reinvesting the funds at different rates, and compensating for negative outflows in the future.

However, we have shown here that mathematically, any investor expecting ex ante to receive cash flows, who actually receives the expected cashflows, achieves the expected YTM/IRR originally calculated, regardless of the reinvestment rate of return. The result arises from the incorporation of maturity and duration into the analysis.

4. The IRR in project evaluation

A final point regarding YTM/IRR van be made in project evaluation. The internal rate of return (IRR) rule is a guideline for deciding whether to proceed with a project or investment. The rule states that a project should be pursued if the internal rate of return is greater than the minimum required rate of return or cost of capital.

The IRR is calculated using the formula [1]. When we evaluate a project or a firm, we make a forecast of different variables to calculate the free cash flow to the firm, usually under the following scheme:

Table 6: Expected cashflows from a project

	t=0	t=1	t=2	t=3	t=4	t=5
Sales		240	330	380	450	520
Operative Expenditures		250	260	270	280	285
Earnings before Interest and Taxes		-10	70	110	170	235
Income Tax		0	21	39	60	82
Net operative profits after Taxes		-10	49	72	111	153
Amortization and Depreciation		30	33	36	40	44
Capital Expenditures		-50	-55	-61	-67	-73
Increase working capital			-9	-5	-7	-7
FCFF (Free cashflow to the firm)		-30	18	42	77	116

We usually cut the forecast at a future point where it becomes useless to continue forecasting, however it is assumed that cashflows continue growing at a steady long rate of growth g, usually lower than the short term growth rate. This works under the assumption of business as an ongoing concern, instead of resorting for instance to the liquidation value (if that were the case, then we shall use the liquidation value of assets as the residual value).

The underlying assumption of such use is that the company or project could be sold at that point for cash at the value obtained from the residual value formula, so it represents a fair price. To mathematically calculate the Residual Value (RV) the following formula applies:

$$\mathrm{RV} = \frac{\mathrm{FF}_{t+1}}{\mathrm{i} - \mathrm{g}} \qquad \textit{Eq 2}$$

Where RV is the present value of a perpetuity cash flow FF_t growing at a constant growth rate g and discounted at a rate i.[2]

Even though extending the cashflows calculation into the future and discounting them or collapsing them into the Residual Value formula provides the same present value of cashflows at a future point (in this case t=5), the effect on the IRR of that assumption is not trivial.

The effect can be shown in the table 7 from the previous exercise, with an assumed discount rate g of 9% and long run rate of growth is 3%.

The first row extends the cashflows in the future by letting them grow at the rate of 3%. The third row shows the cashflows only for the first five periods, and collapses the cashflows afterward under the residual value calculation.

In can be easily seen that discounting at time t=5 future cashflows from t=6 to the future at the 9% discount rate provides the same value as the residual value formula does (i.e. $ 1999). The only difference between cashflows from both rows is how the cashflows are plot.

Table 7: Extended cashflows and cashflows with residual value

	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9	... so on
	Investment	FCFF1	FCFF2	FCFF3	FCFF4	FCFF5	FCFF6	FCFF7	FCFF8	FCFF9	
FCFF ad infinitum	-525	-30	18	42	77	116	120	124	127	131	
Residual @ t=5						1999					
Short FCFF w/residual	-525	-30	18	42	77	2116					

However, though both rows provide the same present value of future cashflows, as we can see in the following calculation, the IRR is much lower in the case where we continue plotting the cashflows into time with respect to the case where we collapse them into the residual value formula

Table 8: Present value and IRR under extended cashflows and cashflows with residual value

	Present value	IRR
Extended Cashlows	$ 1.449,90	16%
Cashflows w/residual	$ 1.449,90	34%

[2] For a constant cash flow, the formula simplifies to CF / i because g is zero. It is the present value of the cash flow stream after the terminal year, which is the last year of the projection period.

So to use the IRR in the second case could be misleading because the investor is not effectively receiving \$ 1999 at t=5 as the residual value, but continues receiving future cashflows afterwards.

The YTM/IRR formula stands for effective cashflows, as we see it is in bonds, where the cashflows arise from the prospect or contract. If we assume a significant cash inflow as we do when we calculate residual value in project evaluation, we could be artificially increasing the IRR due to the previously mentioned effect.

5. Synthesis

The yield to maturity (YTM) or internal rate of return (IRR) arising from cashflows of a bond or an investment project does not require any underlying assumptions. Actually, the calculation of a YTM/IRR is indeed a mathematical calculation, yielding a number with no further interpretations.

There appears to be a semantic confusion between achieving an IRR or yield to maturity and obtaining a target amount of wealth at “maturity”, which is the basis of our approach to distinguish between both.

With a riskless asset, a rate of return YTM/IRR is obtained regardless of reinvesting or not any cash flows. If “maturity” is defined as duration, a strategy aimed at accumulating wealth at the YTM/IRR is possible even if reinvestments are made at a different rate. If rates change frequently, immunisation strategies by keeping the duration of the fund equal to the target horizon will achieve it.

YTM/IRRs derived from certain, promised, and expected cash flows are not directly comparable. YTM/IRRs of uncertain cash flows are rates of return conditional on the cash flows occurring as projected or promised or as expected. Therefore, reinvestment strategies are uncertain.

The distinction applies for project evaluation in corporate finance, where if IRR is calculated from effective cashflow (as it is YTM in bonds, where cashflows arise from a contract) and hence if it is assumed a cash equivalent of future cashflows as residual value, then the financial indicator of return could be misleading.

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